Engineering Notes

Sample Estimates of B-66B Low-Level, Clear-Air Gust Field Parameters

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Nomenclature

b_{11}, b_{12}, b	= activity parameters, fps
P_{11}, P_{12}	= portions of turbulence
Go_{11}, Go_{12}	= number of gusts/mile ≥ 0
U_V, U_L, U_{DE}	= vertical, lateral, derived equivalent gust velocities, fps
$G(U_V \geq X)$, $G(U_L \geq X)$, $G(U_{DE} \geq X)$	= cumulative probability distributions for $U_V \geq X$, $U_L \geq X$, $U_{DE} \geq X$
$\phi(X)$, $\phi(Xi)$	= $\phi(X) = \exp[-\exp(-y)]$, $y = \alpha(X - u)$; $\phi(Xi)$ is probability density distribution for extremal $X \leq Xi$; α, u are parameters of $\phi(X)$
$F(X)$	= $1 - \phi(X)$
Δn	= c.g. incremental vertical load factor, g's
L	= scale of turbulence, ft
h	= pressure altitude, ft
$f(L)$	= sample estimate for probability distribution of L
χ^2	= chi-squared distribution

BECAUSE of differences in gust field parameters from different sources,[1-3] and current loads studies on several transport aircraft with low-level mission design requirements, we have recently estimated low-level gust field parameters for the samples of 42 data runs analyzed in Ref. 4. They were estimated from initial and extremal cumulative probability distributions without reference to the distribution of standard deviations of the gust velocities of the primary data samples. The parameters for vertical and lateral gust field velocities differ significantly for these data (which is not surprising, considering boundary effects), requiring two subsets of parameters for each field: subset 1, subscript 11, from the sample initial cumulative probability distribution; subset 2, subscript 12, from the extremal cumulative probability distribution. The first subscript of each subset denotes the type of gust field, clear-air turbulence in this case. The second subscript denotes the subset: 1 initial, 2 for extremal.

Estimates of sample gust field parameters for low-level, clear-air vertical and lateral gust fields are given in Table 1. These estimates are from vertical $U_{V(+)}$ and lateral $U_{L(+,-)}$ initial and extremal distributions and are conservative of the samples of (+), (−), or (+, −) data analyzed. Sample estimates for the expected number of gusts per mile $\geq X$ for U_V and U_L gust velocities, dropping (+), (−), or (+, −) direction signs are

Table 1 Estimates of sample vertical and lateral gust field parameters

Vertical Gust Field, true b's		
Activity parameters	$b_{11} = 2.72$	$b_{12} = 4.31$
Portions of turbulence	$P_{11} = 1$	$P_{12} = 1$
No. of gusts/mile ≥ 0	$Go_{11} = 13.93$	$Go_{12} = 0.6$
Laterial gust field, true b's	...	...
Activity parameters	$b_{11} = 3.01$	$b_{12} = 5.49$
Portions of turbulence	$P_{11} = 1$	$P_{12} = 1$
No. of gusts/mile ≥ 0	$Go_{11} = 11.01$	$Go_{12} = 0.33$

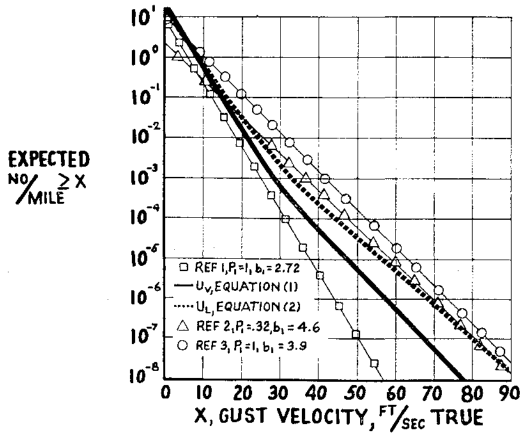

Fig. 1 Low-level, clear-air turbulence cumulative distributions.

$$G(U_V \geq X) = 13.93 \exp(-X/2.72) + 0.60 \times \exp(-X/4.31) \ldots \quad (1)$$

$$G(U_L \geq X) = 11.01 \exp(-X/3.01) + 0.33 \times \exp(-X/5.49) \ldots \quad (2)$$

It is of interest to note that sample estimates of $UDE(+)$ gust field parameters for these data give the equation

$$G(U_{DE} \geq X) = 11.92 \exp(-X/2.79) + 0.65 \times \exp(-X/4.39) \ldots \quad (3)$$

Equations (1) and (2) are plotted in Fig. 1 along with distributions for low-level clear-air turbulence from sources noted. For small X, $G(X)$ is asymptotic to the initial distribution. For large X, $G(X)$ is asymptotic to the extremal distribution. In extremal notation the parameter α is equivalent to $1/b$ in gust field notation, since for large X

$$1 - \phi(X) = F(X) \cong \exp(-y)$$
$$= \exp - \alpha(X - u)$$
$$\log F(X) = -\alpha(X - u)$$
$$d \log F(X)/dX = -\alpha$$

On this basis, α is equivalent to $1/b$. Reference 1 distribution is a fit to Ref. 4, Fig. 34, p. 84, $U_{v(+)}$, which is strictly an initial distribution source. References 2 and 3 distributions appear to be for the most part conservative, particularly Ref. 3. However, Ref. 2 is low below 10 fps.

Distributions of primary data samples and extremes of these samples are quite different. The occurrences of large values in primary data samples depend upon the duration of the sample. The shorter the duration, the less likely the occurrence of an extreme. The extremes of relative short samples appear rather frequently over longer durations of sampling, depending upon the size of the extreme, tending to dominate the distribution for large values of the variate. In Ref. 2, it seems that the sample durations were of long duration and that values of the variate $\Delta n < 0.3$ g's were dropped from the sample data, thus making the distribution predominately extremal, but mixed. Judging from the activity parameter, the sample data were predominately extremal. The following considerations may be major sources of unexplained differences in

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reported gust field parameters: sample size, duration of primary data, size of variate, dropped variates, initial distributions, extremal distributions, mixed distributions, method of estimating parameters, etc.

Low-level gusts refer to gusts at low levels of altitude above the terrain, nominally less than 1000 ft, but may be at any pressure altitude depending on atmospheric conditions. Fits to the sample of 42 vertical gust velocity data runs analyzed in Ref. 4, for scale of turbulence and pressure altitude, both linear regression and probability density distribution, are as follows. The linear regression, p. 263 of Ref. 4, is

$$L = 467.827 + 45.274 \times 10^{-3}h \ldots \quad (4)$$

$$0 \leq h \leq 12{,}000 \text{ ft}$$

where L is the scale of turbulence, ft, and h is the pressure altitude, ft. The probability density distribution, p. 264 of Ref. 4, is

$$f(L) = \frac{1}{40.5 \times 10^6} L^3 \exp(-L/150)\ldots \quad (5)$$

where $f(L)$ is χ^2 distributed with mean 600, mode 450, median 550, and variance 90,000.

References

[1] Neuls, G. S., "Optimum fatigue spectra," Aeronautical Systems Division TDR 61-235 (August 1961).

[2] Press, H. and Steiner, R., "An approach to the problem of estimating severe and repeated gust loads for missile operations," NASA TN 4332 (September 1958).

[3] USAF Military Specifications, MIL-A-8866, Aeronautical Standards Group (May 1960).

[4] Saunders, K. D., "B-66B low-level gust study," Vol. 1, *Technical Analysis*, Wright Air Development Division TR 60-305 (March 1961).

Optimum Spacing of Shell Frames

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Nomenclature

E_f = modulus of elasticity of frame, psi
I_f = moment of inertia of frame, in.^{4}
M = bending moment, in.-lb
R = radius of shell, in.
t_e = effective shell thickness, in.
L = frame spacing, in.
A_f = area of frame, in.^{2}
N = maximum axial loading, lb/in.
f_c = applied compressive stress, psi
F_c = allowable compressive stress, psi
γ_s = density of shell material, lb/in.^{3}
γ_f = density of frame material, lb/in.^{3}
k = shape factor
c = column fixity coefficient
K = shape factor

In Reference (1), Shanley develops an expression for the combined weight of covering and frame material per inch of fuselage length. He shows, graphically, that a frame spacing exists which yields the minimum weight of total structure. For many types of structures the allowable compressive stress may be approximated analytically in terms of loading; e.g., the curves in Fig. 1, illustrating structure manufactured in currently used materials, can be approximated by two straight lines. Using this approximation it is shown that the optimum frame spacing may be readily estimated.

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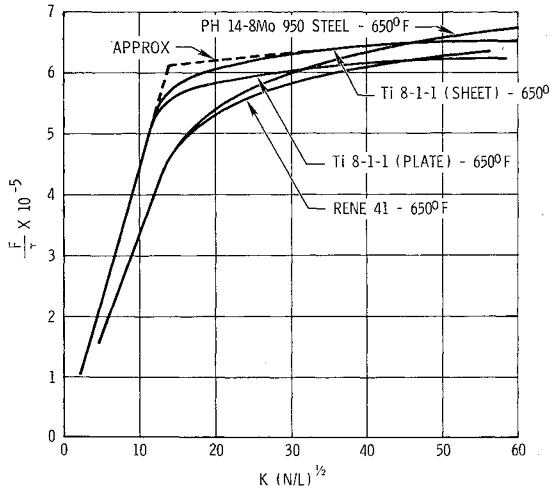

Fig. 1 Allowable compressive stress curves, approximated in terms of loading.

According to Ref. (1) the stiffness of the frame section required to prevent general instability is

$$I_f = \frac{4C_f R^2 M}{E_f L} \quad (1)$$

and

$$I_f = kA_f^2 \quad (2)$$

Substituting A_f from Eq. (2) into Eq. (1), solving for A_f, and dividing by L,

$$\frac{A_f}{L} = \frac{2RM^{1/2}}{L^{3/2}} \left(\frac{C_f}{kE_f}\right)^{1/2} \quad (3)$$

Then the combined weight of the shell and frame per inch of fuselage length may be written

$$W = 2\pi R t_e \gamma_s + \frac{4\pi R^2 M^{1/2} \gamma_f}{L^{3/2}} \left(\frac{C_f}{kE_f}\right)^{1/2} \quad (4)$$

Now

$$N = f_c t_e = \frac{M}{\pi R^2} \quad (5)$$

Solving for t_e from Eq. (5) and substituting into Eq. (4),

$$W = \frac{2\pi R N \gamma_s}{f_c} + \frac{4\pi^{3/2} R^3 C_f c^{3/4} \gamma_f}{NkE_f^{1/2}} (N/L')^{3/2} \quad (6)$$

where $L' = L/(c)^{1/2}$ The allowable compressive stress in the surface can be approximated by a linear function of $(N/L')^{1/2}$:

$$F_c = A(N/L')^{1/2} \qquad (N/L')^{1/2} < \psi$$
$$F_c = B + H(N/L')^{1/2} \qquad (N/L')^{1/2} > \psi \quad (7)$$

Substituting F_c from Eq. (7) into Eq. (6), we obtain

$$W = \frac{2\pi R N \gamma_s}{B + H(N/L')^{1/2}} + \frac{4\pi^{3/2} R^3 C_f^{1/2} \gamma_f}{k^{1/2} c^{3/4} E_f^{1/2}} (N/L')^{3/2} \quad (8)$$

Letting $\alpha = (N/L')^{1/2}$, Eq. (8) becomes

$$W = [J/(B + H\alpha)] + P\alpha^3 \quad (9)$$

The value of α corresponding to the minimum value of W is obtained by differentiating W with regard to α, setting equal to zero, and solving for α:

$$\frac{dW}{d\alpha} = -\frac{JH}{(B + H\alpha)^2} + 3P\alpha^2 \quad (10)$$